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COST BENEFIT ANALYSIS OF A TWO UNIT COLD STANDBY SYSTEM WITH CORRELATED FAILURES AND REPAIRS AND INSPECTION TIME

[1*]P.K. Tyagi, [2]R.K. Agarwal

[1]Department of Statistics, DPBS P.G. College Anoopshahr-203390, India

[2]Department of Mathematics, DPBS P.G. College Anoopshar-203390, India

ABSTRACT: *This paper presents the analysis of a two - unit cold standby system with inspection time. There are two repairmen i.e. expert and assistant repairman. The failure and repair times of each unit have been assumed to be correlated and their joint density has taken as a bivariate exponential. Using the regenerative point technique, various reliability characteristics of interest have been obtained. The behavior of MSTF has been also studied graphically.*

KEYWORDS: *Mean time to system failure (MTSF), Inspection time, Regenerative point technique, Cold standby system, Availability Analysis.*

INTRODUCTION

Several authors including references [1-3] and [6-8] in the field of reliability theory have analyzed two unit-cold standby systems assuming that there is only single repair facility to repair a failed unit. As soon as an operative unit fails, its repair starts instantaneously by a repairman. This assumptions not hold in many real-life systems. Sometimes it has been observed that system manager has two repairmen i.e. expert and assistant repairman. As soon as operative unit fails, the expert repairman instructs to the assistant repairman what tools and equipments are needed to repair the failed unit. After inspection of the failed unit, assistant repairman starts the repair of the failed unit. In all the above systems [1-5], it has been assumed that failure and repair times are uncorrelated. But, in practical life, it has been also observed that there are some systems [6-8] with correlated failure and repair time.

Keeping these facts in view, we have analyzed here a two-unit cold standby system with correlated failure and repair time and inspection time.

* **Corresponding Author: Dr. P. K. Tyagi**
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MODEL DESCRIPTION

The system consists of two identical units. Initially one unit is operative and the other is cold standby. Upon failure of operative unit, the standby unit is put into operation instantaneously and the expert repairman instructs to the assistant repairman what tools and equipments are needed to repair the failed unit. After inspection, the failed unit is taken up for repair by assistant repairman. A repaired unit works like a new one. The system fails completely if during inspection time and during the repair of a failed unit, the other unit also fails. When failed unit is under inspection or under repair, the other failed unit waits for inspection. The inspection time distribution is negative exponential with parameter θ. The joint distribution of failure (X) and repair (Y) times of each unit is bivariate exponential with respective parameters (α,β).

NOTATION AND STATES

In order to define the states of the system, we define the following symbols,

N_0/N_s :unit is in normal mode and operative / standby.

FI/FwI/FI_c: failed unit is under inspection/waits for inspection/ inspection is continued from earlier state.

Fr/Frc: failed unit is under repair/ repair continued from earlier state.

With these symbols, the possible states of the system are as follows:

UP States: $\underline{S_0}$=(N_0,N_s), $\underline{S_1}$=(FI,N_0) $\underline{S_2}$=(F_r,N_0)

Failed States: S_3=(Frc,FwI), S_4=(FIc,FwI) $\underline{S_5}$=(Fr,FwI)

The underlined states are regenerative states. A transition diagram of the system model is shown in Fig 1.

TRANSITION PROBABILITIES AND SOJOURN TIMES

We obtain the following non-zero elements of the transition probability matrix P:

$$p_{01} = 1, p_{12} = \theta/[\theta + \alpha\,(1 - r)], p_{15}^{(4)} = \alpha(1 - r\ )/[\theta + \alpha(1 - r)],$$

$$p_{20} = \beta\,/[\alpha + \beta], \quad p_{21}^{(3)} = \alpha\,/[\alpha + \beta]\ and\ p_{51} = 1$$

Clearly, the above probabilities satisfy the following relations:

$$p_{01} = 1, p_{12} + p_{15}^{(4)} = 1,\ p_{20} + p_{21}^{(3)} = 1\ \ and p_{51} = 1$$

The mean sojourn times in various states are:

$$T_0 = 1/\alpha(1-r), T_1 = 1/[\theta + \alpha(1-r)], T_2 = 1/[(\alpha+\beta)(1-r),$$

$$T_3 = T_5 = 1\,/\beta(1-r)\,, T_4 = 1/[\theta]$$

MEAN TIME TO SYSTEM FAILURE

Considering the failed states S_3, S_4 and S_5 as absorbing, we have by simple probabilistic reasoning, the following recurrence relations:

$$\pi_0(t) = Q_{01}(t) s\, \pi_1(t) \qquad (1)$$

$$\pi_1(t) = Q_{12}(t) s\, \pi_2(t) + Q_{14}(t) \qquad (2)$$

$$\pi_2(t) = Q_{20}(t) s\, \pi_0(t) + Q_{23}(t) \qquad (3)$$

Taking the Laplace-Stieltjes transform of (1)- (3) and using the well known formula for MTSF, we get

$$\text{MTSF} = (T_0 + T_1 + p_{12}T_2\ )/(1 - p_{12}p_{20}) \qquad (4)$$

AVAILABILITY ANALYSIS

From the arguments used in the theory of regenerative processes, we observe:

$$A_0(t) = M_0(t) + q_{01}(t) © A_1(t) \qquad (5)$$

$$A_1(t) = M_1(t) + q_{12}(t) © A_2(t) + q_{15}^{(4)}(t) © A_5(t) \qquad (6)$$

$$A_2(t) = M_2(t) + q_{20}(t) © A_0(t) + q_{21}^{(3)}(t) © A_1(t) \qquad (7)$$

$$A_5(t) = q_{51}(t) © A_1(t) \qquad (8)$$

Where,

$$M_0(t) = \exp[-\alpha(1-r)t], M_1(t) = \exp[-\{\theta + \alpha(1-r)\}t]$$

And $M_2(t) = \exp[-(\alpha + \beta)(1-r)t]$

By taking the laplace transform of (5)-(9), we can obtain $A_0^*(s)$, Using this result, the steady-state availability of the system is

$$A_0 = \lim_{t\to\infty} A_0\,(t) = \lim_{s\to 0} sA_0^*\,(s) = \frac{N_1}{D} \quad (9)$$

Where,

$$N_1 = (T_0(1 - p_{12}p_{21}^{(3)} - p_{16}^{(4)}) + (T_1 + p_{12}T_2)\ and$$

$$D = T_0 p_{12} p_{20} + (T_1 + p_{15}T_4) + (T_2 + p_{23}T_3) + T_5 p_{15}^4$$

BUSY PERIOD ANALYSIS

Using the definition of $B_i(t)$ and simple probabilistic reasoning, we have the following recursive relations:

$$B_0(t) = q_{01}t \copyright B_1(t) \quad (10)$$

$$B_1(t) = W_1(t) + q_{12}(t) \copyright B_2(t) + q_{15}^{(4)}(t) \copyright B_5(t) \quad (11)$$

$$B_2(t) = W_2(t) + q_{20}(t) \copyright B_0(t) + q_{21}^{(3)}(t) \copyright B_1(t) \quad (12)$$

$$B_5(t) = W_5(t) + q_{51}(t) \copyright B_1(t) \quad (13)$$

Where$W_1(t) = \exp[-\{\theta + \alpha(1 - r_1)\}t]W_2(t) = \exp[-\{(\alpha + \beta)(1 - r)\}t]$

And $W_5(t) = \exp[-\{\beta(1 - r)t]$

By taking Laplace transform of (10) – (13), we can obtain B_0^*(s). The steady state probability that both repairman are busy given by:

$$B_0 = \lim_{t\to\infty} B_0\,(t) = \lim_{s\to 0} sB_0^*\,(s) = \frac{N_2}{D} \quad (14)$$

Where, $N_2 = (T_1 + p_{12}T_2)$

COST ANALYSIS

1. The expected up-time of the system and busy period of repairman in (0, t) are:

$$\mu_{up}(t) = \int_0^t A_0\,(u)du$$

$$\mu_b\,(s) = \int_0^t B_0\,(u)du$$

So that

$$\mu_{up}^{*}\ (s) = \frac{A_{0\ (s)}^{*}}{s}$$

$$\mu_{b}^{*}\ (s) = \frac{B_{0\ (s)}^{*}}{s}$$

Now expected profit incurred in (0,t) is

$$P(t) = c_1\mu_{up}(t) - c_2\mu_b(t) \qquad (15)$$

Where C_1 is the revenue per unit up time by the system and C_2 is the repair cost per unit time.
Therefore the expected profit per unit time in steady state is

$$P = \lim_{t\to\infty}\frac{P(t)}{t} = \lim_{s\to 0} s^2\ P_0^{*}\ (s) = C_1A_0 - C_2B_0 \qquad (16)$$

CONCLUSION

To observe the effect of correlation (r) and inspection rate(θ) on the system performance,We plot the MTSF against correlation and inspection rate keeping the other parameters fixed. The curves are shown in figure 2. From these curve we observe the MTSF increases with increasing in correlation (r) and decreases with increasing in inspection rate (θ). Thus to obtain a high MTSF, we must have a high correlation andsmall inspection rate (θ).Hence we conclude that the system leads to a better overall performance for large value of correlation and small value of inspection rate.

REFERENCES

1. Rander M.C.,Suresh Sharma and Ashok Kumar (1994)., Cost Analysis of two dissimilar cold standby system with preventive maintenance and replacement of standby,Microelectron Reliability 34(1), 171-174.
2. Goel L.R., Rakesh Gupta and P.K. Tyagi (1991), CHE failure in two standby system with slow switch, repair and post repair, Microelectron Reliability 31, 219-222.
3. R. Laxmi, Rakesh Gupta and P.Chaudhary (2021), A two-non identical unit standby system with helping unit of the priority unit, Int. J. Agricult, Stat. 17(2), 873-881.

4. Ashok Kumar, S.K. Gupta and Gulshan Taneja (1996), Comparative Study of the profit of a two server system including patience time and instruction time, Microelectron Reliab. 36(101, 1595-1601).
5. P. Chaudhary and Ankita Sharma (2021), A two identical unit standby system with preventive maintenance and two-phase repair by two repairman, Int J. Agricult. Stat. Sci 17 (2), 699-707.
6. Rakesh Gupta, P.K. Tyagi and Ramkishan (1996), A two-unit system with correlated failures and appearance and disappearance of repairman, Int. J. Syst. Sci.23, 379-391.
7. Rakesh Gupta, Vikas Tyagi and P.K. Tyagi (1997), Cost Benefit analysis of a two-unit standby system with post-repair, activation time and correlated failures and repair,Journal of quality in maintenance engineering 3(1), (55-63).
8. P.K. Tyagi and R.K. Agarwal (2022), Cost analysis of a two dissimilar units with dependent repair time and preparation time, Int.J. Agricut. Stat. Sci 18(1), 311-314.

Transition Diagram

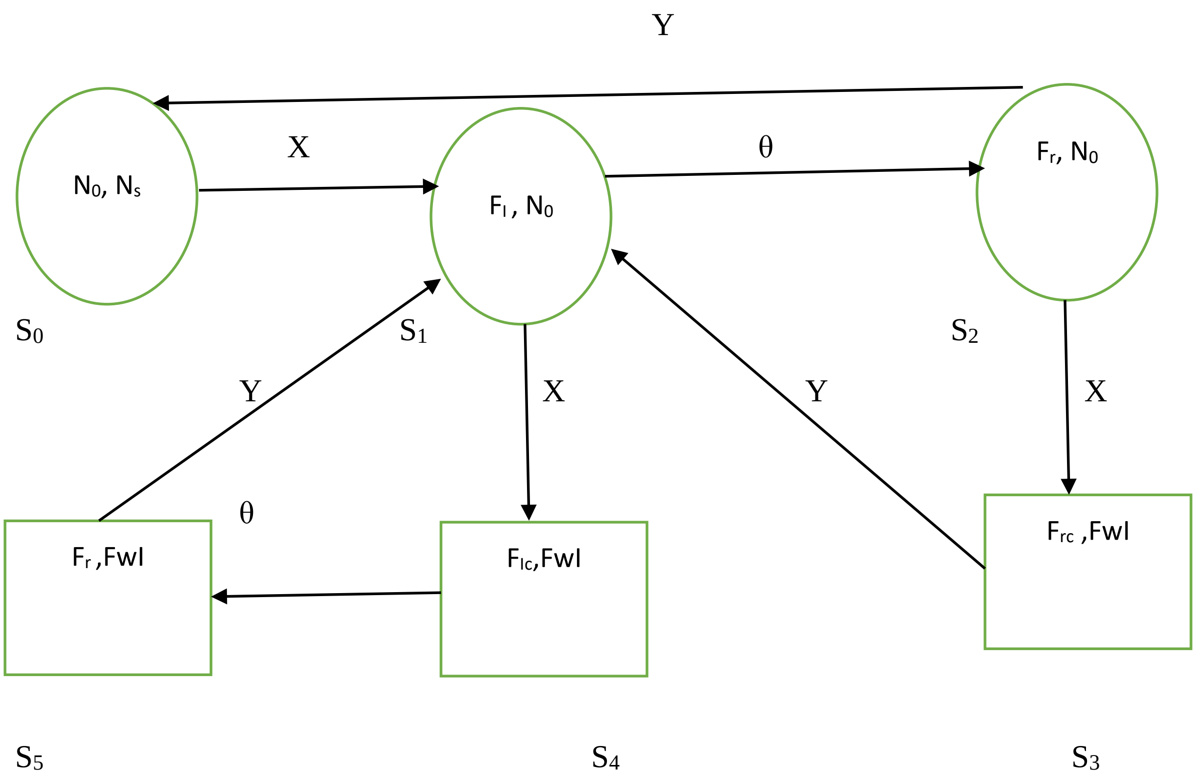
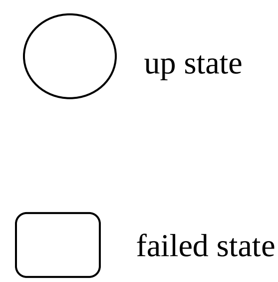


Figure. 1



Behaviour of MTSF w.r.t. to θ for different Values of r.

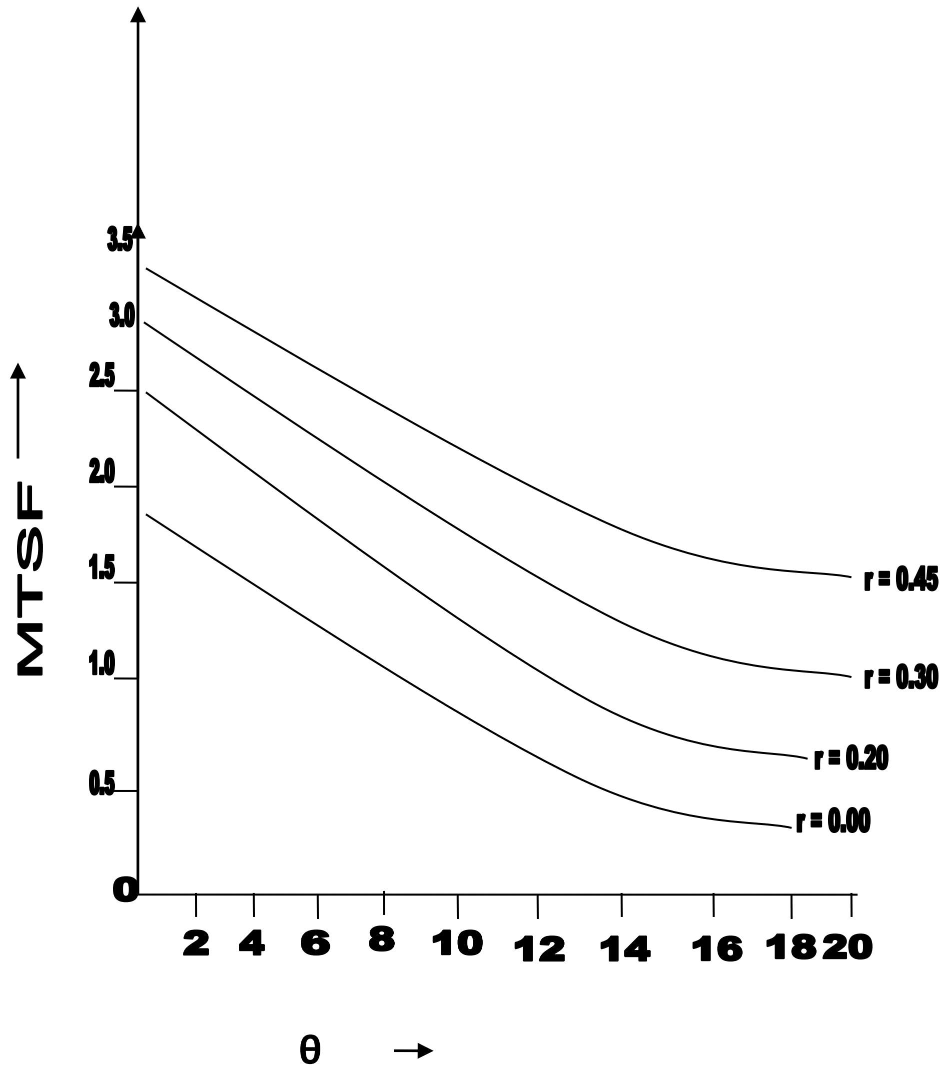


Figure 2